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FRESH WATER LENS PRODUCED BY UNIFORM INFILTRATION

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Iu. I. KAPRANOV
(Novosibirsk)

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The plane model proposed by N. N. Verigin for a stabilized fresh water lens produced by uniform infiltration is investigated in hydrodynamic formulation in the case of equidistant horizontal slit drains. Formulas are obtained for the separation boundary, the depression curve, and characteristic dimesions of the lens.

1. Statement of problem. The considered pattern of flow is shown in Fig. 1. An infinite system of parallel slit drains of the same width $2h$ normal to the xy-plane is disposed along the x-axis (the y-axis is directed vertically upward). We assume that the soil is homogeneous and of unbounded depth, and the distance between the middle of adjacent drains is constant and equal $2L$. Fresh water of density ρ_1 seeps from the surface of the soil over the free boundary (the depression curve AB), passes through the lens (region G), and is drawn off through the drains. Salt ground water of density ρ_2 lies below the separation boundary (curve EF). The case of incomplete flooding of drains is considered and it is assumed that infiltration intensity ε (per unit length of the x-axis) is constant, the ground water is stationary [1, 2], and the motion in the lens is stationary.

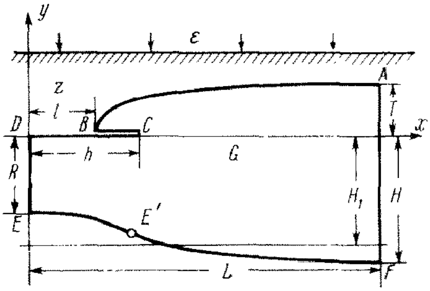

Fig. 1

Investigation of the described model which is periodic with respect to x of period $2L$ and symmetric about the y-axis reduces to the solution of the following mathematical problem [1]. We have to construct region G of the form shown in Fig. 1 with a pair of harmonically conjugate functions φ and ψ inside it, so as to satisfy the boundary conditions

$$(\varphi + y)|_{AB} = [\psi + \varepsilon_0 (L - x)]|_{AB} = \varphi|_{BC,\,CD} = \tag{1.1}$$
$$\psi|_{DE,\,EF,\,FA} = (\varphi - \rho_0 y - \varphi_1)|_{EF} = 0$$
$$\rho_0 = (\rho_2 - \rho_1) / \rho_1, \qquad \varepsilon_0 = \varepsilon / k$$

where φ and ψ are, respectively, the velocity potential and the stream function normalized with respect to the ground filtration coefficient k, and φ_1 is some constant. In what follows the complex potential is denoted by $\omega = \varphi + i\psi$, and $z = x + iy$ denotes any arbitrary point of region G.

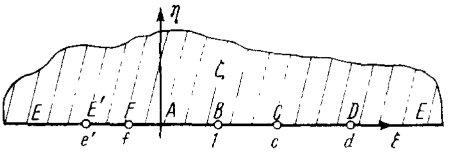

Fig. 2

We introduce the auxilliary complex variable $\zeta = \xi + i\eta$ and functions $z(\zeta)$ which conformally maps the upper half-plane $\operatorname{Im} \zeta = \eta > 0$ onto region G (the correspondence of points is shown in Fig. 2), and the complex velocity $w = u - iv = d\omega / dz$ and

$$Z(\zeta) = dz / d\zeta, \tag{1.2}$$
$$\Omega(\zeta) = d\omega / d\zeta$$

2. Construction of function $w(\zeta)$. The hodograph region which corresponds to boundary conditions (1.1) [1.3] is shown in Fig. 3. Using the transformation

$$W = \ln \frac{2i + \beta w}{2i\sigma - \alpha w} \tag{2.1}$$
$$\sigma = \frac{\sqrt{\rho_0 + \varepsilon_0} + \sqrt{\varepsilon_0 (1 + \rho_0)}}{\sqrt{\rho_0 + \varepsilon_0} - \sqrt{\varepsilon_0 (1 + \rho_0)}},$$
$$\alpha, \beta = \frac{\sigma - 1}{\varepsilon_0} \pm (\sigma + 1)$$

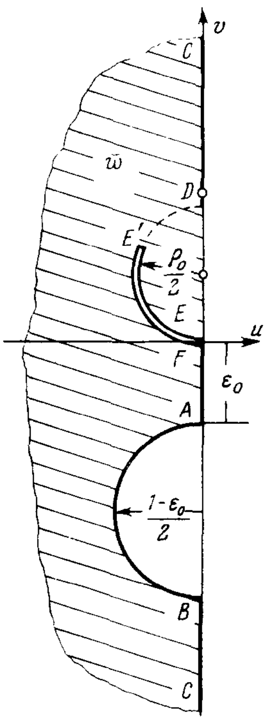

Fig. 3

it can be converted into a horizontal half-band of width π with a vertical slit (Fig. 4). Using the Schwarz-Christoffel formula for function W we obtain

$$\frac{dW}{d\zeta} = c_0 \Phi(\zeta) = c_0 \frac{\zeta - e'}{\zeta - q} \Phi_0(\zeta), \tag{2.2}$$
$$\Phi_0 = [(\zeta - f)\,\zeta\,(\zeta - 1)]^{-1/2}$$

where c_0 is the unknown constant and $\zeta = q$ is the inverse

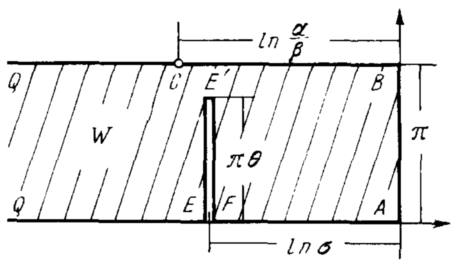

Fig. 4

image of an infinitely distant point of the half-band relative to the image $W = W(\zeta)$.

In the W-plane (Fig. 4) the half-band width and the length of segments FA and BC are specified. It is also known that the distances between points F, E' and E, E' are the same. Hence parameters c_0, f, q, e' and c must satisfy equations

$$c_0(q-e')\,\Phi_0(q) = 1, \quad \ln\sigma = c_0\int_f^0 |\Phi|\,ds \tag{2.3}$$

$$\ln\frac{\alpha}{\beta} = c_0\int_1^c |\Phi|\,ds, \quad \int_{e'}^{f} |\Phi|\,ds = \int_{-\infty}^{e'} |\Phi|\,ds$$

(the branch Φ_0 is chosen so that $\arg\Phi_0 = 0$ for $1 < \zeta = \xi < +\infty$).

Investigation of this system is conveniently carried out with the substitution of the following new quantities:

$$\lambda^2 = \frac{-f}{1-f}, \quad \mu^2 = \frac{1-f}{q-f}, \quad \nu = \frac{q-e'}{q-f}, \quad \delta = \frac{1-f}{c-f} \tag{2.4}$$
$$(0 < \lambda,\ \mu < 1; \quad \mu^2 < \delta < 1)$$

for parameters f, q, e' and c This reduces the first equation of system (2.3) to the form

$$c_0 = \frac{1}{\mu\nu}\sqrt{\frac{1-\mu^2}{1-\lambda^2}(1-\lambda^2\mu^2)} \tag{2.5}$$

while the last, after some simple transformations, determines the dependence

$$\nu = \nu(\lambda,\mu) = (1-\lambda^2\mu^2)K(\lambda')\,[K(\lambda') - \lambda^2\mu^2\Pi(\lambda^2\mu^2 - \mu - 1, \lambda')], \quad \lambda' = \sqrt{1-\lambda^2} \tag{2.6}$$

where $K(\lambda)$ and $\Pi(\mu,\lambda)$ are complete elliptic integrals of the first and third kind, respectively [4].

It follows from the first two equations of system (2.3) that

$$\int_f^0 |\Phi_0|\,ds = (q-e')\left[\int_f^0 |\Phi_0|\,ds/(q-s) - \ln\sigma \,/\sqrt{(q-f)\,q\,(q-1)}\right]$$

By substituting $t = 1 - s/f$ for the variable of integration and (2.6) for ν we obtain

$$K(\lambda)\,[K(\lambda') - \lambda^2\mu^2\Pi(\lambda^2\mu^2 - 1, \lambda')] =$$
$$(1-\lambda^2\mu^2)\,K(\lambda')\left[\Pi(-\lambda^2\mu^2,\lambda) - \frac{\mu}{2}\ln\sigma/\sqrt{(1-\mu^2)(1-\lambda^2\mu^2)}\right]$$

It remains to express the complete elliptic integrals Π of the third kind by incomplete elliptic integrals of the first and second kind and then apply Legendre's relationship [4]. As the result, we obtain the relationship

$$\mu = \mathrm{sn}\,(K(\lambda) - K(\lambda')\ln\sigma/\pi), \quad \lambda_0 < \lambda < 1 \tag{2.7}$$

where sn denotes the elliptic sine modulo λ, and λ_0 is determined by the equation

$$\ln\sigma/\pi = K(\lambda_0)/K(\lambda_0'), \quad 0 < \lambda_0 < 1 \tag{2.8}$$

To determine parameter δ it is sufficient to use (2.4) and (2.5), and rewrite the last of remaining equations of system (2.3) in the form

$$\ln\frac{\alpha}{\beta} = c_0(\nu-1)\sqrt{1-\lambda^2}\int_\delta^1 \frac{s+\mu^2/(\nu-1)}{s-\mu^2}\,\frac{ds}{\sqrt{s(1-s)(1-\lambda^2 s)}} \tag{2.9}$$

3. Construction of functions $z(\zeta)$ and $\omega(\zeta)$. The analytical theory of linear differential equations, first applied for solving filtration problems in [5], is used here. According to that theory real values of "accessory" parameters β_0, β_1 and a pair V_1, V_2 of linearly independent solutions of the equation

$$V'' + a(\zeta)V' + b(\zeta)V = 0 \tag{3.1}$$
$$a(\zeta) = \frac{-1}{\zeta - e'} + \frac{1/2}{\zeta - f} + \frac{1/2}{\zeta} + \frac{1/2}{\zeta - 1}$$
$$b(\zeta) = (\beta_0 + \beta_1\zeta)[(\zeta - e')(\zeta - f)\zeta(\zeta - 1)]^{-1}$$

with singular points $\zeta = e'$, f, 0, 1 and ∞, such that

$$w(\zeta) = V_1(\zeta) / V_2(\zeta) \tag{3.2}$$

can be found.

By virtue of the relationship

$$V_1'V_2 - V_1V_2' = \exp[-\textstyle\int a d\zeta] \text{ const}$$

which is valid for any two linearly independent solutions V_1 and V_2 of Eq. (3.1), the latter implies that $w'V_2^2 = \text{const}(\zeta - e')[(\zeta - f)\zeta(\zeta - 1)]^{-1/2}$

Substituting expressions (2.1) and (2.2) into the left-hand part of this equality, we first determine V_2, and then, from (3.2) obtain V_1. It follows from (1.2) and (3.2) that $\Omega / V_1 = Z / V_2 = f(\zeta)$, where $f(\zeta)$ is some function which can be determined by comparing the singularities of functions Ω and Z obtained from the boundary conditions (1.1) with those of solutions V_1 and V_2. Such comparison shows that the product $f(\zeta)\sqrt{(\zeta - f)\zeta(\zeta - 1)(\zeta - d)}$ is regular and bounded throughout the closed plane ζ. Consequently it is identically equal to a constant, hence Ω and Z are determined.

Now, using (1.2), it is possible to write expressions for the unknown functions $\omega(\zeta)$ and $z(\zeta)$

$$\omega = -\rho_0 R + \varphi_1 + c_1\int_\infty^\zeta \left[\sigma F - \frac{1}{F}\right] g\, d\zeta \tag{3.3}$$

$$z = -iR - i\frac{c_1}{2}\int_\infty^\zeta \left[\alpha F + \frac{\beta}{F}\right] g\, d\zeta$$

$$F = \exp\left[\frac{1}{2}W(\zeta)\right], \quad g = \left[\frac{\zeta - q}{(\zeta - f)\zeta(\zeta - 1)(\zeta - d)}\right]^{1/2}$$

where c_1 denotes the unknown constant, function $W(\zeta)$ is defined in Sect. 2, and branch g is chosen so as to satisfy the condition $\arg g = 0$ for $\max(d, g) < \zeta = \xi < +\infty$.

The characteristic dimensions of the lens (Fig. 1) are: l, the drain width $2h$, the lens width $2L$, the maximum height T of the depression curve, and the maximum and minimum distances H and R, respectively, of the separation boundary from the level of drains. For these quantities we obtain from (2.3) the following expressions:

$$L = \frac{c_1}{2\sqrt{\sigma}}(\sigma\beta + \alpha)\int_{-\infty}^{f} \cos[f_4(\xi)]\,|g(\xi)|\,d\xi \tag{3.4}$$

$$T = \frac{c_1}{2}(\alpha - \beta)\int_0^1 \cos[f_2(\xi)]\,|g(\xi)|\,d\xi$$

$$H = -T + \frac{c_1}{2}\int_f^0 \{\alpha \exp[-f_3(\xi)] + \beta \exp[f_3(\xi)]\}\,|g(\xi)|\,d\xi$$

$$R = H - \frac{c_1}{2\sqrt{\sigma}}(\sigma\beta - \alpha)\int_{-\infty}^{f} \sin[f_4(\xi)]\,|g(\xi)|\,d\xi$$

$$l = L - \frac{c_1}{2}(\alpha + \beta)\int_0^1 \sin[f_2(\xi)]\,|g(\xi)|\,d\xi$$

$$h = l + \frac{c_1}{2}\int_1^c \{\alpha \exp[-f_1(\xi)] - \beta \exp[f_1(\xi)]\}\,|g(\xi)|\,d\xi$$

where $g(\zeta)$ is taken from (3.3) and functions $f_1(\xi), \ldots, f_4(\xi)$ which are determined with the use of (2.2) along segments $(1, c)$, $(0, 1)$, $(f, 0)$ and $(-\infty, f)$, respectively, are defined by formulas

$$f_1(\xi) = \frac{c_0}{2}\int_1^\xi |\Phi|\,ds, \qquad f_2(\xi) = \frac{c_0}{2}\int_\xi^1 |\Phi|\,ds \tag{3.5}$$

$$f_3(\xi) = \frac{c_0}{2}\int_\xi^0 |\Phi|\,ds, \qquad f_4(\xi) = \frac{c_0}{2}\int_{-\infty}^{\xi} \frac{e' - s}{q - s}\,|\Phi_0|\,ds$$

It was shown in Sect. 2 that q, e', c and c_0 are uniquely defined by the specified value of parameter f. Hence for the determination of characteristic dimensions of the lens in (3.4) it is sufficient to specify parameters f, d and $c_1 > 0$ only. (Expression of the kind of (3.4) can be obtained for the constant φ_1 which appears in boundary conditions (1.1)).

In this notation the separation boundary EF (Fig. 1) which in the ζ-plane corresponds to the real semiaxis $\xi < f$ assumes the form

$$x = x_2(\xi) \equiv \frac{c_1}{2\sqrt{\sigma}}(\sigma\beta + \alpha)\int_{-\infty}^{\xi} \cos[f_4(s)]\,|g(s)|\,ds \tag{3.6}$$

$$y = y_2(\xi) \equiv -R - \frac{c_1}{2\sqrt{\sigma}}(\sigma\beta - \alpha)\int_{-\infty}^{\xi} \sin[f_4(s)]\,|g(s)|\,ds$$

It follows from (2.2) and (2.3) that function $f_4(\xi)$ monotonically increases from zero to $\pi\theta/2 < \pi/2$ (Fig. 4) when ξ changes from $-\infty$ to e', and when ξ approaches f it tends monotonically to zero. This, owing to (3.6) means that $x_2'(\xi) > 0$ and $y_2'(\xi) < 0$. Hence the separation boundary can be specified by the equation $y = y_2(x)$, where $y_2(x)$ monotonically decreases from $y_2(0) = -R$ to $y_2(L) = -H$, with dy_2/dx vanishing at points $x = 0$ and $x = L$ the depression curve AB (Fig. 1) is the image of segment $0 < \xi < 1$ of the real axis of the ζ-plane for mapping $z(\zeta)$ and is defined by formula

$$x = x_1(\xi) \equiv l + \frac{c_1}{2}(\alpha + \beta)\int_\xi^1 \sin[f_2(s)]\,|g(s)|\,ds \tag{3.7}$$

$$y = y_1(\xi) \equiv \frac{c_1}{2}(\alpha - \beta)\int_{\xi}^{1} \cos[f_2(s)]\,|g(s)|\,ds$$

The considerations in Sect. 2 imply that $f_2(\xi)$ monotonically decreases from zero to $\pi/2$, with ξ increasing from zero to unity. This in turn implies that $x_1'(\xi) < 0$ and $y_1'(\xi) < 0$, hence it is possible to use the equation $y = y_1(x)$, which monotonically increases from $y_1(0) = l$ to $y_1(L) = T$, instead of (3.7), with $y_1'(l) = +\infty$ and $y_1'(L) = 0$.

It is now possible to determine the distance from the drain to the initial (horizontal) ground water table $y = -H_1$ (Fig. 1). For this we denote by $(x_0, -H_1)$ the point of intersection of the separation boundary $y = y_2(x)$ with the straight line $y = -H_1$. If S_1 is the area enclosed between curves $y = y_2(x)$ and $y = -H_1$ for $0 < x < x_0$, and S_2 is the similar area for $x_0 < x < L$, then the equality $S_1 = S_2$ must be satisfied. From this we obtain the sought expression

$$H_1 = -\int_0^L \frac{y_2(x)\,dx}{L} \tag{3.8}$$

4. Mapping parameters, limit cases. In the direct formulation mapping parameters f, d and c_1 are the unknowns. For their determination it is sufficient to specify any three characteristic dimensions of the lens. Here the lens width $2L$, the drain width $2h$, and the distance H_1 between the drain and the unperturbed level of ground water have been chosen for these. Then for given f and d the first of expressions in (3.3) evidently determines $c_1 > 0$. The latter plays the part of the modulus of expansion and can be eliminated by normalizing all dimensions of the lens with respect to its half-width

$$l^\circ = \frac{l}{L},\quad h^\circ = \frac{h}{L},\quad T^\circ = \frac{T}{L},\quad R^\circ = \frac{R}{L},\quad H^\circ = \frac{H}{L},\quad H_1^\circ = \frac{H_1}{L}$$

Below we use parameters (2.4) and substitute

$$\tau^2 = \frac{1-f}{d-f},\quad 0 < \tau^2 < \delta$$

for d.

Formulas (2.4) – (2.9), (3.4), (3.8) and (3.6) yield the system of equations $H_1^\circ = H_1^\circ(\lambda, \tau)$, $h^\circ = h^\circ(\lambda, \tau)$ which will be used for determining λ and τ.

The difficulty of the analytical investigation of this system is due not only to the complexity of functions $H_1^\circ(\lambda, \tau)$ and $h^\circ(\lambda, \tau)$, but also to the necessity of investigating the limitations which in the plane H_1°, h° define the region of applicability of the considered flow pattern. The first of these limitations is the consequence of the motion symmetry about the y-axis and is expressed by the inequality $l > 0$. The second stipulates that the maximum height T of the depression curve must not exceed the distance from the ground level to the plane of drains. The third limitation is less evident, and will be dealt with below.

The mapping parameters λ and τ were determined by numerical methods. For this ε_0 and ρ_0 were specified, and λ_0 was determined by (2.8). Then λ was fixed in the interval $(\lambda_0, 1)$ and μ, ν, c_0 and δ were calculated by (2.5) – (2.7) and (2.9), respectively. The latter were substituted into (3.4) and (3.8), the subdivision of interval $(0, \delta, (\lambda))$ was decided upon and the characteristic dimensions of the lens were calculated for each τ^2 of the subdivision. A similar procedure was carried out for various values of λ. The calculations had shown that $H_1^\circ(\lambda, \tau)$ monotonically increases with increasing τ. This made it possible to determine the inverse function $\tau = \tau(\lambda, H_1^0)$ and investigate the relationship $h^\circ(\lambda; H_1^\circ) = h^\circ(\lambda, \tau(\lambda, H_1^\circ))$. The latter proved to be monotonically increasing with λ and it became possible to determine λ for specified h° and H_1°.

The effect of the drain width on the nature of change of the lens dimensions is illustrated by the following example. Let $\rho_0 = 0.01$, $\varepsilon_0 = 0.1$, $H_1^\circ = 1$ and h° runs through 0.6, 0.7, 0.8 and 0.9. The calculations yielded: $T_0 = 0.0427$, 0.0312, 0.0204 and 0.0101; $R^\circ = 0.7034$, 0.8737, 0.9497 and 0.9868; $H^\circ = 1.1408$, 1.0912, 1.0522 and 1.0134; $l^\circ = 0.5969$, 0.6946, 0.7995 and 0.8997, respectively. It will be seen that with increasing h° parameter T° decreases, while R° approaches H° and tends to H_1° (a similar behavior is observed for other values of parameters ρ_0, ε_0 and H_1^0). Analytical investigation had shown that $\mu \to 1$, $\nu \to 2$, $\delta \to 1$, $h^\circ \to 1$ and $T^\circ \to 0$, when $\lambda \to 1$, with the separation boundary becoming the straight line $y = y_2(x) = -H_1 = -LK(\tau)/K(\tau')$. Thus this limit is characterized by the linking of adjacent drains, which results in the disappearance of the ground water bulge. The lower part of the lens is now completely isolated from infiltration, motion inside it ceases, and the separation line flattens out and reverts to its initial position. In the above example the value of λ is close to λ_0 defined in (2.8) when $h^\circ = 0{,}6$.

It can be shown that when $\lambda \to \lambda_0$, then $\mu \to 0$, $\nu \to 1$, δ tends to the completely determined value $\delta(\lambda_0)$, and the length of the slit $EE'F$ in the W-plane (Fig. 4) tends to π. At the limit the point of inflection E' (Fig. 1) merges with point E which becomes a cusp, and in the hodograph plane the semicircle $|\overline{w} - \rho_0/2| < \rho_0/2$ "falls out". Parameter h° then tends to some value h_1° which is the minimum admissible for the considered pattern of flow, and this represents the third limitation mentioned earlier. From the physical point of view h_1° may be considered the critical value of the drain width for specified ρ_0, ε_0 and H_1°, since for $h^\circ < h_1^\circ$ the drain cannot cope with the drainage of infiltrating water. When $h^\circ = h_1^\circ$ the intensity of filtration is at its maximum throughout the range $h_1^\circ < h^\circ < 1$, and the rate of filtration $Q = \varepsilon_0(1 - l^\circ)kL$ reaches maximum, while the drain width is minimum. Calculations show that h_1° decreases with increasing H_1°; for example, for $\rho_0 = 0.01$, $\varepsilon_0 = 0.1$ and $H_1^\circ = 1.4$ the critical width h_1° is close to 0.39.

It can be shown that for $\lambda = \lambda_0$ and $\tau \to 0$, the cusp E reaches the drain: $R^\circ \to 0$. The equations of the separation boundary and of the depression curve become, respectively,

$$y = -H\sqrt{1 - (1 - x/L)^2}, \quad 0 < x < L$$

$$y = T\sqrt{1 - (1 - x/L)^2/(1 - l/L)^2}, \quad l < x < L$$

This limit case was previously considered in [7].

A further characteristic property follows from calculations: for fixed ρ_0, H_1° and h° the remainder $h^\circ - l^\circ$ increases with increasing ε_0. Thus, for example, for $\rho_0 = 0.01$, $H_1^\circ = 1$ and $h^\circ = 0.8$ the values $\varepsilon_0 = 0.1$, 0.5 and 0.7 correspond to $h^\circ - l^\circ = 0.0005$, 0.0318 and 0.081. From the physical point of view such behavior is completely natural.

Let us point out one more limit case of the considered problem, namely $\rho_0 \to \infty$.

It follows from (2.1), (3.4) and (3.7) that in that case $R^\circ - H^\circ \to 0$ and the separation boundary becomes the straight line $y = y_2(x) = -H$, i.e. that boundary becomes an impermeable base.

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OPTIMUM CONTROL OF THE FORCED MOTIONS OF SYSTEMS WITH CONTINUOUSLY DISTRIBUTED PARAMETERS

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L. A. FIL'SHTINSKII
(Novosibirsk)

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We propose one of the possible versions of the optimum control of the forced motions of elastic systems of the type of rods, plates, and shells. We apply the procedure developed to elementary problems on the transition of a freely-supported rod or plate from an initial state φ, ψ to the rest state in the least possible time T in the presence of a constraint on the forcing load. We use the elementary results of theory of the l-problem of moments of Krein [1 - 3].

1. We consider a hinge-supported rod undergoing forced motions under the action of a load $f(x, t)$. The complete system of equations defining the state of the rod at any instant t has the form

$$\frac{\partial^4 w}{\partial x^4} + \frac{\rho F}{EJ}\frac{\partial^2 w}{\partial t^2} = \frac{f(x,t)}{EJ}, \quad x \in (0, l), \quad t > 0$$
$$w(0,t) = w(l,t) = 0, \quad w_{xx}(0,t) = w_{xx}(l,t) = 0 \tag{1.1}$$
$$w(x, 0) = \varphi(x), \; w^\cdot(x, 0) = \psi(x), \; w^\cdot = \partial w / \partial t$$